## LadHyX Seminar – November 29, 10:45

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## Nonlinear instabilities in immiscible viscous multilayer flows

Simple shear flows such as plane Couette flows are known to be linearly stable for all Reynolds numbers. If more than one immiscible fluids are present there can an interfacial instability that produces traveling waves or more complex nonlinear dynamics such as spatiotemporal chaos. The instabilities require non-zero Reynolds numbers and have been reported in experiments. Part of this talk will describe these using multiscale asymptotic analysis and agreement with both direct numerical simulations and experiments is very good. When multiple layers are present there are now at least two free interfaces. Asymptotic solutions will be presented that yield a system of coupled partial differential equations for the interfacial positions. The equations are parabolic with fourth order diffusion when surface tension is present or second order diffusion when surface tension is absent and the fluids are stably stratified. The equations generically support instabilities even at zero Reynolds numbers. These emerge physically from an interaction between the interfaces and manifest themselves mathematically through hyperbolic to elliptic transitions of the fluxes of the equations. We use the theory of  $2x^2$  systems of conservation laws to postulate a nonlinear stability criterion that can tell us whether a system which is initially linearly stable (i.e. the initial conditions are in the hyperbolic region of the flux function) can (i) become nonlinearly unstable, i.e. a large enough initial condition produces a large time nonlinear response, or (ii) remains nonlinearly stable, i.e. the solution decays to zero irrespective of the initial amplitude of the perturbation. Time permitting, we will consider fully nonlinear deformations in the large surface tension limit to derive coupled Benney type equations. Their fluxes also support hyperbolic-elliptic transitions and numerical solutions will be described giving rise to intricate nonlinear traveling waves. Transitional flows are harder to find than in the weakly nonlinear models, but examples will be given where linearly stable initial conditions transition into elliptic regions to sustain energy growth and saturation to nonlinear states.